



Name:

### Achilles and the Tortoise (Zeno's Paradox)

The Tortoise challenged Achilles to a race, claiming that he would win as long as Achilles gave him a small head start. Achilles laughed at this, for of course he was a mighty warrior and swift of foot, whereas the Tortoise was heavy and slow.

"How big a head start do you need?" he asked the Tortoise with a smile.

"Ten meters," the latter replied.

Achilles laughed louder than ever. "You will surely lose, my friend, in that case," he told the Tortoise, "but let us race, if you wish it."

"On the contrary," said the Tortoise, "I will win, and I can prove it to you by a simple argument."

"Go on then," Achilles replied, with less confidence than he felt before. He knew he was the superior athlete, but he also knew the Tortoise had the sharper wits, and he had lost many a bewildering argument with him before this.

"Suppose," began the Tortoise, "that you give me a 10-meter head start. Would you say that you could cover that 10 meters between us very quickly?"

"Very quickly," Achilles affirmed.

"And in that time, how far should I have gone, do you think?"

"Perhaps a meter – no more," said Achilles after a moment's thought.

"Very well," replied the Tortoise, "so now there is a meter between us. And you would catch up that distance very quickly?"

"Very quickly indeed!"

"And yet, in that time I shall have gone a little way farther, so that now you must catch that distance up, yes?"

"Ye-es," said Achilles slowly.

"And while you are doing so, I shall have gone a little way farther, so that you must then catch up the new distance," the Tortoise continued smoothly.

Achilles said nothing.

"And so you see, in each moment you must be catching up the distance between us, and yet I – at the same time – will be adding a new distance, however small, for you to catch up again."

"Indeed, it must be so," said Achilles wearily.

"And so you can never catch up," the Tortoise concluded sympathetically.

"You are right, as always," said Achilles sadly – and conceded the race.

Imagine that Achilles (a name from Greek mythology) does decide to run the race against the Tortoise. Suppose that he runs at a constant speed of **600 feet per minute**, while the tortoise walks at a constant speed of **100 feet per minute**. And suppose that Achilles gives the Tortoise a head start of **2000 feet**.

- a) Write an equation for the distance (in feet) that Achilles runs **A(t)** as a function of time **t** (in minutes).

Write an equation for the distance (feet) that the Tortoise walks **T(t)** as a function of time **t** (minutes), with a head start of **2000 feet**.

Write and simplify an equation for the distance between them as a function of time **t** using function notation **A(t)** and **T(t)**.

- b) Use calculus notation and algebra to write and determine the limit of the distance between Achilles and the Tortoise as **t** approaches **4**.